THE RELATIONSHIP BETWEEN A BRAHMANIC FIRE ALTAR AND A SOLAR FORMULA IN ANGKOR WAT

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ABSTRACT. A fire altar described in the *Śatapatha Brāhmana* (circa 2000 BCE) was built in five layers representing the universe. The top one, denoting the progress of the Sun along the ecliptic, was a circle of 29 bricks, the southeast quadrant containing eight bricks and the others seven. In the temple of Angkor Wat (12th century CE), which is recognized to have been laid out according to an astronomical plan, the dimensions of the rectangular central gallery are 176.37 hat one way and 189 hat the other. The disposition of bricks in the fire altar appears to be related to the dimensions of the gallery.

RÉSUMÉ. Un autel du feu décrit dans le texte *Śatapatha Brāhmana* (datant de 2000 avant J.C.) a été construit en cinq niveau qui représentent l'univers. Le niveaux supérieur, qui représente la trajectoire du Soleil le long de l' écliptique, est un cercle de 29 briques dont le quadrant sud-est contient huit briques et les autres, sept briques chaque. Dans le temple d'Angkor Wat (12e siècle), que l'on reconnait avoir été conçu selon un plan astronomique, les dimensions de la galerie centrale rectangulaire sont de 176,37 hat dans une direction et de 189 hat dans l'autre. Il semblerait y avoir un rapport entre la disposition des briques de l'autel du feu et les dimensions de la galerie.
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1. INTRODUCTION

In a pair of recent papers, Kak (1998a, 1998b) has discussed the astronomical implications of a fire altar described in Book 8 of the *Śatapatha Brāhmana*, a Vedic (ancient Sanskrit) text that may date from as early as 2000 BCE. It is not certain that the altar, or others of the kind (Kak 1995), was ever constructed, but its design has interesting implications. The altar as planned consisted of five layers of bricks that denoted the layers of the universe spanning from the Earth to the Sun. The layout of the top layer for the altar is illustrated in figure 1, from which it is seen that the altar was rimmed by a circle of twenty-nine bricks. The bricks appear to denote the annual course of the Sun along the ecliptic. Three of the four quadrants contain seven bricks each, but the southeast (spring) quadrant contains eight, the two extra bricks representing two halves of a standard-sized brick, with one half-brick placed just after the spring equinox and the other just before the summer solstice. The altar appears to represent one of the earliest descriptions of the varying rate of motion of the Sun along the ecliptic.

Astronomy was prized as the greatest of sciences in ancient India. Our understanding of its cultural importance on the Indian subcontinent and of the relationship between Indian and Western astronomy is, at present, undergoing radical changes as a consequence of the study of pre-Babylonian Indian astronomy of the Brāhmanic era (circa 1000 BCE). Indian astronomical ideas of that era were intertwined with ritual, and that made the astronomical traditions very conservative. We demonstrate here that the *Śatapatha Brāhmana* concept of the varying rate of motion of the Sun is also symbolized in the architecture of the famous abandoned temple complex of Angkor Wat in Cambodia. The latter was built during the reign of the Khmer Emperor Suryavarman II, who ruled from 1130 to 1150 CE, and its architecture and sculptural decoration clearly reflect a Hindu influence. Excellent popular books on Angkor Wat are available

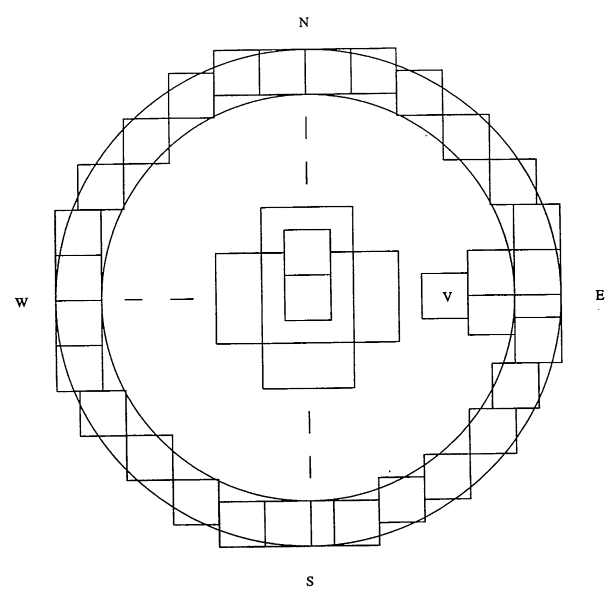

FIG. 1 — The topmost layer of the *Śatapatha* altar describing the circuit of the Sun (from Kak 1998b).

for interested readers. The work of Grolier & Artaud (1957) is recommended for its outstanding rotogravure illustrations. Coèdes (1963) has written a history of the builder and a description of the complex, and has shown that it was intended as a mausoleum.

The temples built by the Khmer kings functioned as focal points for the local cultural devotion to the Devaraja, and were at the same time earthly and symbolic representations of the mythical Mount Meru, which represents the cosmological home of the Hindu gods and the axis of the world system. An innovation introduced by the Khmer Emperor Jayavarman, who preceded Suryavarman II, was the adoption of Vedic symbols and other traditions with Indian roots. Such modifications help to explain why the concepts imbedded in the *Brāhmana* may be found expressed in the architectural features of Angkor Wat. The word Angkor itself derives from the Sanskrit word, "*Nagara*", which stands for city — the centre of the religious and secular cosmos.

Stencel *et al.* (1973, hereinafter SGM) identified several recognizable astronomical alignments in Angkor Wat. By making use of a measured survey of the temple done by Nafilyan and published in 1969, and by converting the measurements into their equivalents in Cambodian cubits or hat (= 0.435 m), they demonstrated that certain dimensions of the temple causeway symbolized the yuga cycles of Hindu cosmology. The temple complex was apparently designed as an observatory, since SGM were able to identify twenty-two alignments in the temple survey that correspond to seasonally important observational sight lines for the rising of the Sun and the Moon. According to Chou Te-kuan, a Chinese merchant who visited Angkor in 1296, just as in China "... there are people who understand astronomy and can calculate the eclipses of the Sun and Moon." The temple was situated in the midst of an extensive irrigation system that stored water from the monsoon flood. It seems likely that one of the responsibilities of the "people" was calendar keeping, which was necessary for anticipating the annual flood and its recession.

In the central tower, the topmost elevation (SGM Table 2, item 6, and figure 7) had external dimensions of 189.00 hat in the east-west direction and 176.37 hat in the north-south direction, corresponding to a semi-perimeter of 365.37 hat. In the words of SGM, it was "perhaps the most outstanding number" in the complex, corresponding as it does to "almost the exact length of the solar year" in days. Presented here is a likely explanation for the numbers as inferred from considerations of the illustration for the fifth layer of the Brāhmanic fire altar.

2. Outline of the Analysis

As a reflection of the annual motion of the Earth in its orbit about the Sun, the Sun appears to move relative to the fixed stars upon a path called the ecliptic, which is a great circle on the celestial sphere. Its angular position along the ecliptic relative to the vernal equinox, which serves as a reference point for celestial co-ordinates, is referred to as ecliptic longitude (or celestial longitude), and is measured eastward from the vernal equinox. Since the orbit of the Earth about the Sun is elliptical rather than circular, the speed of the Earth in its orbit varies with its distance from the Sun. It moves fastest near perihelion at the beginning of January and slowest near aphelion at the beginning of July. At such times the apparent speed of the Sun along the ecliptic is correspondingly faster and slower relative to the average rate.

A clue to the formula that appears to have been used to calculate the ratio of the dimensions in the gallery of Angkor Wat, *i.e.* 189.00/176.37, is to be found in a paper by Åaboe (1974). Around 100 BCE, Babylonian astronomers calculated the ecliptic longitude of the Sun by a rule called Babylonian System A. They assumed that the Sun travelled along the ecliptic at constant, but unequal, speeds in two separate sectors. The duration of the two sectors, in days, corresponds very closely to the dimensions of the gallery at Angkor Wat. Since the Angkor numbers can also explain the features of the fire altar described in the *Śatapatha Brāhmana*, it can be speculated that perhaps the Babylonians borrowed from Hindu predecessors.

The ancient Hindus had derived a remarkably accurate value for the length of the mean lunar month. By implication they must have had available to them several centuries of day-counts between various lunar phases in order to obtain a reliable value for the synodic month — the cycle of the Moon's phases. It seems that in all likelihood they would have discovered that the mean duration of the lunar month is also variable in its day-count on a seasonal basis, from which it would have been inferred that the angular speed of the Sun along the ecliptic is itself a function of the season of the year. Calculations for the mean day-count for different seasons would have been found to converge to minimum and maximum values, which occurred (as noted above) at times near perihelion and aphelion respectively. From that they could have formulated an algorithm for describing the motion of the Sun along the ecliptic. If the respective speeds were adopted to be constant over half-year intervals (of 6 moons plus $5^1/_2$ days), the results would be a duration of 176.37 days for the seasonal sector that included winter solstice and a duration of 189 days for the seasonal sector that included summer solstice.

3. The Astronomical Roots of the Temple Dimensions

According to Kepler's Laws of planetary motion, planets move in elliptical orbits about the Sun with the Sun occupying one of the two foci of the ellipse, and the radius vector from the Sun to a planet sweeps out equal areas of orbit in equal time intervals. In Newtonian mechanics such properties are readily derived from the laws of gravitation applied to central force problems and are paraphrased to read that the orbit of one object about another is described by a conic section, and orbital angular momentum is conserved.

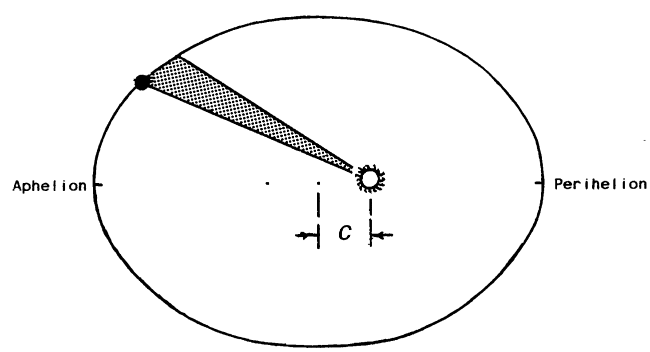

Fig. 2 — An illustration of Kepler's Second Law depicting the area swept out by the radius vector from the Sun. This distance *c* is the centre distance for the ellipse.

A typical elliptical orbit is illustrated in figure 2, which illustrates the terms inherent to Kepler's Second Law — the law of equal areas. Since the Second Law is simply an expression of the conservation of angular momentum in a closed system, it is equivalent to the expression

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4. The Brāhmanic System of Observation

Although the ancients must have developed an arithmetic capable of forming means or averages, they had not yet formulated the concept of irrational numbers. Fractions were therefore expressed as ratios of integers. The arithmetic may have been clumsy, but they used it effectively.

Modern positional astronomy requires the use of exceedingly precise clocks. For the ancients, the only clocks available were day-counts taken from observations of the cycles of the Moon. Over the course of a year, because of the eccentricity of the Earth's orbit, a lunar month varies in length. However, day-counts for the lunar month are not affected by the tilt of the Earth's axis.

It is convenient to consider successive syzygies for the Moon rather than complete months of the Moon's phases. Syzygy refers to the times of either Full Moon or New Moon, *i.e.* to the alignment of the Earth and the Moon with the Sun. Between syzygies the nominal half-month averages 14.765 days, and there are 24.737 of them per annum. Because of the large eccentricity of the Moon's orbit, the half-cycle representing New Moon to Full Moon is in general not equal to the half-cycle representing Full Moon to New Moon. At first the ancients would have counted the days between syzygies without regard to type, and may have continued to do so out of conservatism.

As may be shown (although not simply), thirty-four centuries ago the spring quarter, equinox to solstice, contained 93.26 days, on average amounting to six half-months plus 4.67 days. The days elapsed from the date of the equinox to the first syzygy for the Moon would vary from year to year, and would be distributed randomly up to just under the maximum possible half-month of 14.765 days. The same was true for the days from the last syzygy of the quarter to the solstice. If we consider the situation to be symmetric about the midspring node, the initial and final waiting intervals must be identical, corresponding to a quarter month of about seven days. The two periods appear to be represented in the fire altar by the half bricks, one placed just after the equinox and the other just before the solstice. The remaining six full bricks correspond to the number of syzygies in the quarter.

Our conclusions are elucidated further by consideration of two possible situations. If the delay after the equinox were 0 to 4 days, six half-months would be completed in the quarter, and the residue to the solstice would be 4 to 0 days. If the initial delay were 5 to 14 days, however, only five half-months would be completed, and the residue to the solstice would be 14 to 5 days. In either case there was symmetry in the mean number of waiting days.

After a century or two, observers would have discovered that the averages, Full Moon to Full Moon, converged better than the averages of the half-month. We refer to observations of the Full Moon rather than of the New Moon, because the New Moon, occurring very close to the Sun in the sky, could be observed less easily. The Earth's orbital velocity reaches a stationary point as it passes through perihelion or aphelion. Thirty-four centuries ago the Sun passed through those points in midautumn and midspring. The velocity of the apparent Sun varied by only a small amount over the two or three weeks before or after reaching those points. Over a few centuries the regression along the ecliptic is only a few degrees, so the average day-count per month would change very little over that time. According to the evidence presented by the layout of the bricks in the altar, we argue that the ancient priests discovered the two stationary points. The average day-count of a month containing either point would converge to the true angular velocity of the Sun along the ecliptic, or rather its reciprocal. It is very reasonable to believe that the marvellous accuracy that the ancients achieved in the ratio 189/176.37 resulted from averaging over centuries of observation.

Although the ancient priests observed the inequality of the half-months, they had no way of observing the continuous change of the Moon's velocity, thinking of it as constant between syzygies. Perhaps they assumed that the Sun behaved likewise, moving at constant but unequal speeds over each half-year.

We can now speculate about the distribution of bricks in the circle of the altar, seven in each of three quadrants and eight in the spring quadrant. The total, 29, differs from the number of half-months per annum, which is properly almost 25. We suggest that the priests retained a tradition of observing by the half-month, reflected in the circle of bricks. The four extras, one per quadrant, must have been added to comply with their system of arithmetic, in which ratios could only be expressed as a ratio of integers. The ratio of the midautumn and midspring day-counts would have to be expressible as a ratio of whole numbers. With one brick added per quadrant, the desired ratio was expressed — the half-year containing spring had 15 bricks, and the other half-year 14. Each brick stood for 12.595 days in the anomalistic year of the epoch. Note that 15×12.595 days = 188.9 days, and 14×12.595 days = 176.3 days. Once again we obtain the significant numbers seen in the dimensions of the central tower at Angkor Wat. There appears to be a symbolism to the layout of the bricks in the fire altar of the *Śatapatha Brāhmana*.

5. Conclusions

As argued here, the ancient Hindus observed the number of days per month over centuries, noticing that the averages converged to different constants for the seasons bracketing midspring and midautumn. The constants are virtually identical to the angular velocities of the Sun along the ecliptic at aphelion and perihelion, respectively. We suggest that the savants constructed an algorithm describing the motion of the Sun along the ecliptic. It moved for half a year with one velocity and half a year with the other, yielding a count of 189 days for the half-year centred on midspring, and 176.37 days for the half-year centred on midautumn. In a fire altar described in the *Śatapatha Brāhmana*, the arrangement of bricks in the fifth and topmost layer, symbolizing the motion of the Sun, appears to depict the same algorithm. The same numbers are found in the dimensions of the central tower of Angkor Wat, and are mentioned in the *Śatapatha Brāhmana*. Apparently, the Indian astronomical concept of a varying rate of motion for the Sun was later adopted by the Babylonians and independently handed down to the astronomers of Angkor Wat.

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$$m\,v\,r = \text{constant}\ ,$$

where m is the mass of the orbiting object, v is its orbital velocity, and r is its distance from the orbited object. For the orbit of the Earth about the Sun, it follows that the Earth's orbital velocity v is inversely proportional to its distance from the Sun. The properties of ellipses are also well known, and the distances from the focal point of an ellipse to the end points of the semi-major axis can be found geometrically. They are:

$$r_{\text{perihelion}} = a(1-e)$$

and

$$r_{\text{aphelion}} = a(1+e)\ ,$$

where a is the length of the semi-major axis of the ellipse and e is the eccentricity of the ellipse = c/a, where c is the centre distance — the distance of the centre of the ellipse from either focus. It follows from the above relationships that the ratio of the velocity of the Earth at perihelion to its velocity at aphelion is expressed as

$$\frac{v_{\text{perihelion}}}{v_{\text{aphelion}}} = \frac{r_{\text{aphelion}}}{r_{\text{perihelion}}} = \frac{a(1+e)}{a(1-e)} = \frac{(1+e)}{(1-e)}\ .$$

The rate at which an elliptical orbit is swept out, however, is given by a separate expression given by

$$\frac{1}{2}r^2\frac{\mathrm{d}\theta}{\mathrm{d}t} = \frac{1}{2}h\ ,$$

where h is the areal constant and $\mathrm{d}\theta/\mathrm{d}t$ is the angular velocity of the planet in its orbit. Clearly, the rate of *angular velocity* for a planet is inversely proportional to the *square* of its distance from the Sun. Thus, for the Earth

$$\frac{\left(\frac{\mathrm{d}\theta}{\mathrm{d}t}\right)_{\text{perihelion}}}{\left(\frac{\mathrm{d}\theta}{\mathrm{d}t}\right)_{\text{aphelion}}} = \frac{r^2_{\text{aphelion}}}{r^2_{\text{perihelion}}} = \frac{a^2(1+e)^2}{a^2(1-e)^2} = \frac{(1+e)^2}{(1-e)^2}\ .$$

It is the angular velocity of the Earth that is tied to the duration of the seasons, since that value determines the rate at which the Sun appears to move along the ecliptic. The accepted value for the present eccentricity of the Earth's orbit is $e = 0.016722$ (Allen 1977). Thus, for the orbit of the Earth about the Sun,

$$\frac{\left(\frac{\mathrm{d}\theta}{\mathrm{d}t}\right)_{\text{perihelion}}}{\left(\frac{\mathrm{d}\theta}{\mathrm{d}t}\right)_{\text{aphelion}}} = \frac{(1+e)^2}{(1-e)^2} = \frac{(1+0.016722)^2}{(1-0.016722)^2} = 1.069182\ .$$

The ratio of the dimensions of the central tower at Angkor Wat is remarkably close to the ratio of the angular velocities at perihelion and aphelion, and is given by

$$\frac{189.00}{176.37} = 1.071611 \approx 1.069182\ !$$

The situation even improves if one considers the temporal dependence of the eccentricity of the Earth's orbit, which is decreasing over the long term. The value was 0.017065 at the time of Angkor Wat's construction, and was 0.017251 in the first century CE. The corresponding values for the ratio of angular velocities for the Earth at perihelion and aphelion are 1.070651 and 1.071474, respectively. The close similarity of the numbers to the ratio of dimensions for the central tower of Angkor Wat seems to be more than coincidental. The two ratios would in fact have been coincident at the beginning of the seventh century BCE.

In our time the seasons are not symmetric about the dates of perihelion and aphelion. For example, at 0^h UT on January 1, 2000, the longitude of the Sun is 280°.460, which follows the 1999 winter solstice (when the Sun's longitude is 270°) by 10°.460. Perihelion follows 0^h January 1 by (360° − 357°.528) = 2°.472. Hence, the date of perihelion at present follows the solstice by 12°.932.

As a result of precession of the equinoxes, however, the co-ordinate system that is used as a reference frame for determining the primary points in the Earth's orbit is actually shifting backwards very slowly relative to the orbit. As well, dynamical effects arising from the gravitational influence of the Sun and the other planets are gradually rotating the Earth's orbit in space relative to an inertial reference frame. The time frame over which the location of perihelion regresses along the ecliptic relative to the equinoxes can be determined as follows. The anomalistic year of 365.259635 days is the time it takes for the Earth to return to the same point in its orbit, whereas the tropical year of 365.242190 days is the time elapsed between consecutive passages of the Sun through the vernal equinox. The difference between the values amounts to 0.017445 day. When converted to a fraction of a tropical year, the value corresponds to a period of 20,937 years, over which the point of perihelion for the Earth's orbit gradually makes a complete regression relative to the equinoxes. The amount of regression amounts to 360°/209.37 centuries = 1°.71946 per century. A shift of 12°.932 (see above) therefore takes 12°.932/1°.71946 century^{-1} = 7.52 centuries, so that perihelion coincided with the winter solstice 7.52 centuries prior to the present era, *i.e.* circa 1250 CE — a century after the construction of Angkor Wat. When perihelion was a further 45° along the ecliptic, it was midway in the autumn sector, and aphelion was midway in the spring sector. That occurred 33.7 centuries before the present, *i.e.* circa 1370 BCE. The latter date coincides roughly with the epoch of the fire altar described in the *Śatapatha Brāhmana*, although the altar may have been planned some time before or afterwards.

With reference to Babylonian System A, we believe that, as an algorithm for the motion of the Sun along the ecliptic, the designers of the altar stipulated that in periods, each of exactly half a year in terms of the mean lunar month, the Sun travelled at two constant but unequal speeds along the ecliptic, consuming 176.37 days in the half year including the winter solstice, and 189 days in the half year including the summer solstice.

Thirty-four centuries ago the duration of the synodic month was 29.530595 days, and the duration of the tropical year was 365.242628 days (Allen 1977). Hence, in the course of a year there were 12 complete cycles of the Moon's phases plus 10.875 days. The ancients would have taken the last number to be 11 days, thereby making the length of the year (29.530595 × 12) + 11 days = 365.367 days long. The number is the same as that appearing in the sum of the dimensions of the tower at Angkor Wat, and appears as well as in the earlier *Śatapatha Brāhmana* (Kak 1998a, 1998b).